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The advantage and disadvantage of implicitly stratified sampling

Peter Lynn

Institute for Social and Economic Research

University of Essex

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Non-Technical Summary

The statistical precision of estimates from sample-based surveys is influenced by the way the sample of persons to be interviewed is selected. “Proportionate stratified sampling” is a technique used to ensure that the sample profile matches that of the population from which the sample is selected in some respects. The better the match between the sample profile and the population profile, the more precise the estimates will be. However, there are many different ways to implement proportionate stratified sampling and a key distinction is between “implicit” and “explicit” proportionate stratified sampling. Implicit stratified sampling would involve, for example, listing all the people in the population in order of date of birth and then sampling every 100th person on the list. Explicit stratified sampling, on the other hand, might involve sorting people into a number of age groups and then randomly sampling 1 in 100 people from each age group in turn.

In this paper we compare these two methods of sample selection. We do this by pretending that the Understanding Society wave 1 sample of respondents is in fact a complete population that we wish to study. Because we know the survey responses of everyone in the population, we can calculate exactly how precise estimates would be with each method of selecting a sample from this population. In this way, we can show how much more precise the implicit method is than the explicit method.

However, one drawback of the implicit method in a real survey situation (when we do not know how all the other population members – not selected into the survey sample – would have answered the survey questions) is that it is not possible to estimate well how precise the survey estimates are. Instead, an approximation is used that tends to under-estimate the precision. Because in this study we know the true precision of the estimates that would be obtained with implicit stratified sampling, we are able to show by how much this precision is under-estimated.

In conclusion, the paper argues that implicit proportionate stratified sampling may be preferable to explicit proportionate stratified sampling as it provides better precision, even though survey users may not have good information about how much more precise the estimates are.

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Abstract

Explicit stratified sampling (ESS) and implicit stratified sampling (ISS) are alternative methods for controlling the distribution of a survey sample, thereby potentially improving the precision of survey estimates. With ESS, unbiased estimation of standard errors is possible, whereas with ISS it is not. Instead, usual practice is to invoke an approximation that tends to result in systematic over-estimation. This can be perceived as a disadvantage of ISS. However, this article demonstrates that true standard errors are smaller with ISS and argues that this advantage may be more important than the ability to obtain unbiased estimates of the standard errors.

Key words: standard error, stratified sampling, survey sampling

JEL classifications: C81, C83

Author contact details: plynn@essex.ac.uk

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1. Introduction

Most surveys use stratified sampling designs. This is done in order to benefit from the precision gains that such designs can bring. For a modest effort in designing the sample, the precision gains can often be equivalent to those that would accrue from carrying out tens or even hundreds of extra interviews. Stratified sampling is therefore highly cost-effective. However, there are many different ways that it can be done. The researcher must choose which variables to use, and how to combine them to define the strata. She must also decide whether all strata should be sampled at the same rate (proportionate stratified sampling) or whether some should be over-sampled, perhaps in order to increase the representation in the sample of certain subgroups (disproportionate stratified sampling). Though the researcher is typically constrained to define strata in terms of information that is either available on the sampling frame or can be linked to the frame, this still usually leaves a lot of options regarding exactly how the information should be used. The better the decisions, the more cost-effective the survey design will be.

This article focuses on one specific decision that the researcher must make: whether to use explicit stratified sampling (ESS) or implicit stratified sampling (ISS). For simplicity, we illustrate the arguments in the context of proportionate stratified sampling, but the arguments apply equally when sampling is disproportionate, as a similar decision must be made within each top-level sampling domain.

ESS involves sorting the population elements into explicit groups (strata) and then selecting a sample independently from each stratum. ISS involves ranking the elements following some ordering principle and then applying systematic sampling, i.e. selecting every nth element. For example, if the sampling frame were a list of people containing a single auxiliary variable, date of birth, proportionate ESS would involve creating strata corresponding to a number of discrete age groups and then selecting, using simple random sampling, a number of people from each group such that the proportion of the sample in each group equals the proportion of the population in the group. ISS, on the other hand, would involve sorting the people from youngest to oldest (or oldest to youngest; this is equivalent) and then selecting every nth person on the list (after generating a random start point).

One advantage of ESS is that it permits different sampling fractions to be applied to different sub-domains of the population (disproportionate stratified sampling), if the strata are created to reflect the sub-domains. But if disproportionate sampling is not desired, this advantage does not apply and it is less clear whether ESS should be used. Another advantage of ESS is that unbiased estimation of the standard errors of survey estimates is possible, provided that the sampling stratum membership is identified on the survey dataset and provided that at least two sample elements are selected from each stratum. With ISS this is not possible and usual practice is to invoke an approximation that tends to result in systematic over-estimation of standard errors. This can be perceived as a disadvantage of ISS. However, this begs the question of whether it is better to know the precision of one's estimates or to have more precise estimates without knowing exactly how much more precise they are.

This article provides an exposition of this distinction between ESS and ISS and attempts, via a simulation study using real survey data, to quantify the extent of the improvement in precision with ISS and the extent of the uncertainty about the improvement in precision if the usual approximation is used to estimate standard errors. In the next section, the relevant aspects of sampling theory are presented and are used to derive an expression for the difference in sampling variance between ESS and ISS. The following sections describe how a simulation study will be used to quantify the true difference in sampling variance between the two designs and the extent to which sampling variance will tend to be over-estimated if the usual approximation is used in the case of ISS. The results from the study are then presented and the implications are discussed in the final section.

2. Sampling Theory

For simplicity of exposition, it will be assumed that survey estimates are means or proportions. It is likely – though not demonstrated in this article – that the findings will be broadly applicable to other types of estimates too. Under ESS, the sampling variance of the sample mean can be expressed (Kish 1965, p.81; Cochrane 1977, p.69) as:

$$\mathrm{Var}(\bar{y}) = \sum_{i=1}^{I} \frac{N_i S_i^2 (N_i - n_i)}{(N^2 n_i)} \qquad \text{- (1)}$$

where $S_i^2 = \mathrm{Var}_i(y_{ik})$ is the variance of y within stratum i (y_{ik} is the value of y for individual k in stratum i);

n_i is the number of sample elements in stratum i;

N_i is the number of population elements in stratum i;

and $N = \sum_{i=1}^{I} N_i$ is the total number of elements in the population.

In this article we will assume the context of proportionate sampling, in which case $\frac{n_i}{N_i} = \frac{n}{N}$, $i = 1, \ldots, I$. With this assumption, expression (1) simplifies to:

$$\mathrm{Var}(\bar{y}) = \frac{\sum_{i=1}^{I} S_i^2 (N_i - n_i)}{nN} \qquad - (2)$$

From this expression it can be seen that differences between strata in terms of y do not contribute to the sampling variance. The sampling variance depends only on the variance of y within the strata. This demonstrates how stratified sampling improves the precision of estimates; by eliminating any influence on the sample of one part of the variance of y, namely the part that is between-strata. Once a survey has been carried out, $\mathrm{Var}(\hat{y})$ can be estimated in a straight-forward manner from the survey data, by substituting the observed within-stratum sample variances (s_i^2) for the corresponding population variances (S_i^2), thus:

$$\widehat{\mathrm{Var}}(\bar{y}) = \frac{\sum_{i=1}^{I} s_i^2 (N_i - n_i)}{nN} \qquad - (3)$$

For ISS designs there is of course no concept of explicit strata, so the {i} in expression (2) are not defined. The design-based variance of a sample mean is equivalent to that under cluster sampling with a sample size of one cluster (Madow and Madow 1944). Unbiased sample-based estimators of this variance do not exist. While a number of estimators have been proposed, all of them are biased and all will over-estimate the variance whenever the stratification effect is anything more than negligible (Wolter 1985, pp.258-262). A commonly-used variance estimation method is to treat the ordered list of selected elements as if each consecutive pair had been selected from the same stratum (Wolter, 1985, pp.250-251). Thus, a systematic sample of n elements from an implicitly-stratified list is treated as if it consisted of simple random samples of size 2 from each of n/2 explicit strata. In order to compare the sampling variance of ISS and ESS, we can consider the situation in which the ISS pseudo-strata are subsets of the ESS strata. This is a realistic reflection of the example mentioned in the previous section of stratifying either explicitly or implicitly using date of birth. We will denote the ISS substrata by j = 1, … , Ji. Then, the approximation usually invoked to estimate the sampling variance associated with ISS is:

$$\widehat{\mathrm{Var}}(\bar{y}) = \frac{\sum_{i=1}^{I}\sum_{j=1}^{J_i} s_j^2 (N_j - n_j)}{nN} \qquad \text{- (4)}$$

whereas the true ISS sampling variance is:

$$\mathrm{Var}(\bar{y}) = \frac{\sum_{h=1}^{N/n}(\bar{y}_h - \bar{\bar{y}})}{(N/n - 1)} \qquad \text{- (5)}$$

where

there are N/n possible samples that could be selected, corresponding to the N/n possible random start points;

$\bar{y}_h$ is the sample mean of y for sample h;

$\bar{\bar{y}} = \frac{n}{N}\sum_{h=1}^{N/n}\bar{y}_h$ is the mean of the N/n sample means.

This can be thought of as the sampling variance of a mean under cluster sampling, with a sample size of one cluster, where the population is divided into N/n clusters, $\bar{y}_h$ are the cluster means, and $\bar{\bar{y}}$ is the population mean.

3. Simulation Methodology

Data from wave 1 of *Understanding Society, the UK Household Longitudinal Study*, are treated as population data. These data are used to calculate the sampling variance of means and proportions under simple random sampling, ESS and ISS, in ways that will be described in this section. *Understanding Society* is a large nationally-representative multi-topic general population survey. A stratified, multi-stage sample of addresses was selected (Lynn, 2009) and all persons aged 16 or over resident at a sample address were eligible for an individual interview at wave 1. Members of ethnic minority groups and residents of Northern Ireland were sampled at higher rates than the remainder of the population. Data collection took place face-to-face in respondent's homes using computer-assisted personal interviewing (CAPI) between January 2009 and March 2011. At wave 1, 50,295 individual interviews were completed with sample members. For the illustrative purposes of this article, these individuals are treated as a population from which survey samples are to be selected.

A set of eleven target parameters were selected for study. Of these, five are means of continuous variables and six are proportions based on binary variables. For each, we are interested in comparing the sampling variance of the sample statistic under alternative sampling designs and the estimate of the ISS sampling variance using the successive pairing approach. For ease of exposition and calculation, for each parameter we first amend the population such that N is a multiple of 100. This allows the subsequent creation of equal-sized explicit strata (each containing $N_i = 100$ elements) and the application of implicitly stratified systematic sampling designs in which the sampling interval takes the integer value of 50, the convenience of which will be explained below. From the 50,295 elements, we first drop any with item missing values. This is done separately for each of the eleven target variables, so the dropped elements will differ between the eleven simulated populations. Then, a further set of *m* elements are dropped (*m* between 0 and 99) in order to round the population size down to a multiple of 100. The *m* elements with the smallest analysis weights (largest inclusion probabilities) are chosen. Descriptive statistics regarding this process are presented in Table 1.

For each estimate, the variance and estimated variance for samples of size $N/_{50}$ will be compared under different designs. The following sub-sections describe the sampling variance metrics that were calculated for each of the eleven parameters to be estimated. All but one of the metrics rely on knowledge of the population size, *N*, and the population variance of *y*, S^2, each of which were derived in the usual way from the population simulated as described above.

3.1 Simple Random Sampling

The variance of $\bar{y}$ under simple random sampling is computed as a benchmark and will be used later in the calculation of design effects for the various sample designs under consideration, to help with interpretation of the findings. It is calculated in the usual way:

$$Var_{SRS}(\bar{y}) = \frac{S^2(N-n)}{nN} \qquad \text{- (6)}$$

Table 1: Simulated Populations for 11 Parameter Estimates

	Understanding Society sample size	Item missing	Also dropped (smallest weights	Simulated population size, N	Sample size, n
Continuous variables					
Total monthly income	50,295	78	17	50,200	1,004
Monthly benefit income	50,295	3,236	59	47,000	940
Number of children	50,295	50	45	50,200	1,004
Hours of sleep	50,295	12,420	75	37,800	756
Body mass index	50,295	6,432	63	43,800	876
Binary variables					
Limiting long-term illness (%)	50,295	0	95	50,200	1,004
Arthritis (%)	50,295	3,234	61	47,000	940
In paid employment (%)	50,295	90	5	50,200	1,004
Has degree (%)	50,295	86	9	50,200	1,004
Lives with spouse/partner (%)	50,295	0	95	50,200	1,004
Religion makes a great difference (%)	50,295	3,234	61	47,000	940

Note: Hours of sleep was asked in a supplemental self-completion questionnaire that was returned by only 85.9% of interview respondents, whereas all other items were administered in the face-to-face interview. The items on body mass index, arthritis and religion were not included in the proxy version of the face-to-face interview, which was administered for 6.4% of respondents.

3.2 Explicit Stratified Sampling with 11 Strata

The first stratified design considered is one with eleven explicit strata, defined by the person's age. The first stratum consists of persons aged 16 to 19; the following nine strata consist of five-year age bands from 20-24 to 60-64; the final stratum consists of person 65 years old or older. Proportionate stratified sampling with a sampling fraction of 1 in 50 is used. The sampling variance of a mean is therefore calculated as in expression (2) above, with $n_i = {N_i}/{50}$ and $I = 11$.

3.3 Explicit Stratified Sampling with *N*/100 Strata

The second stratified design considered is one with N/100 equal-sized explicit strata, again defined by the person's age. It can be seen from Table 1 that this corresponds to between 378 and 502 strata. The strata are created by first sorting the population in increasing order of age and then treating the first 100 in sorted order as the first stratum, and so on. A simple random sample of n = 2 is selected from each stratum. The sampling variance of a mean is therefore calculated as in expression (2) above, with $n_i = 2$ and $I = {N}/{100}$.

3.4 Implicit Stratified Sampling with *n* = *N*/50

The third design considered involves sorting the population in increasing order of age and then selecting a systematic random sample of *N*/50 cases using a random start between 1 and *N*/50. There are therefore *N*/50 possible samples that could be selected and the sampling variance of a mean is calculated as the variance of the *N*/50 corresponding sample means, as in expression (5), with *n* = 50.

In addition to calculating the true sampling variance for this design, the expected value of the estimated sampling variance was calculated using the consecutive pairs method outlined in section 2 above. This was done by calculating the estimate produced by expression (5) for each of the *N*/50 possible samples and then taking the mean of these *N*/50 values.

4. RESULTS

For each of the eleven variables, Table 2 presents the true standard error of the sample mean under each of the four sample designs under consideration, as well as the expected value of the estimate of the standard error for the ISS design under the consecutive pairs method. The true value of the population mean is also presented for reference (first column). It is worth noting firstly that the relative standard errors vary greatly between the eleven estimates. Under SRS, they range from 0.01 to 0.08, with the exception of body mass index, which has a relative standard error of 0.65 (driven by a number of influential outliers). This provides a range of circumstances in which to compare the effects of alternative stratified sample designs.

As expected, standard errors are in all cases smaller under stratified sampling that under simple random sampling. In fact the rank order of the four designs in terms of standard error is the same for all eleven estimates: ESS with eleven strata provides an improvement in precision over SRS, ESS with around 500 strata (N/100) provides a further improvement, and ISS improves precision further still. The relative extent of the standard error reduction varies between the estimates, however. For example, for estimating mean number of children or the proportion of people in paid employment most of the gains to be had from stratifying by age accrue with the use of just eleven explicit strata: extensions to 500 strata or ISS provide only very modest marginal gains. For body mass index and for the proportion suffering from arthritis, on the other hand, the gains in moving from eleven to 500 explicit strata are similar or greater in magnitude to those in moving from no strata (SRS) to eleven.

These differences evidently reflect the differing nature of the associations of the variables with age and are illustrated in Figure 1, which presents the design effect for each of the three stratified designs (ratio of sampling variance under ESS or ISS to that under SRS). The proportion suffering from arthritis stands out as the estimate that gains most in terms of precision from each of the successive enhancements to stratification. The precision gain in moving from the ESS11 to the ESS(N/100) design demonstrates that tendency to suffer from arthritis is quite strongly associated with age, even within eleven strata of the ESS11 design. However, the further gain in moving to the ISS design shows that even within (at least some of) the 470 strata in the ESS(N/100) design there remains an association of arthritis with age. This may seem surprising considering that each of the 470 strata covers an age range of only around 2.5 months, on average, but is explained by the strata towards the upper end of the age range – where arthritis is most prevalent – covering larger age ranges, reflecting the smaller population sizes. The design effect of around 0.65 for this estimate with ISS – the smallest of all the design effects in this study – represents a very considerable precision gain. Without stratification, this improvement in precision would require an increase in the sample size with SRS from 940 to 1,443 – an increase that would have considerable cost.

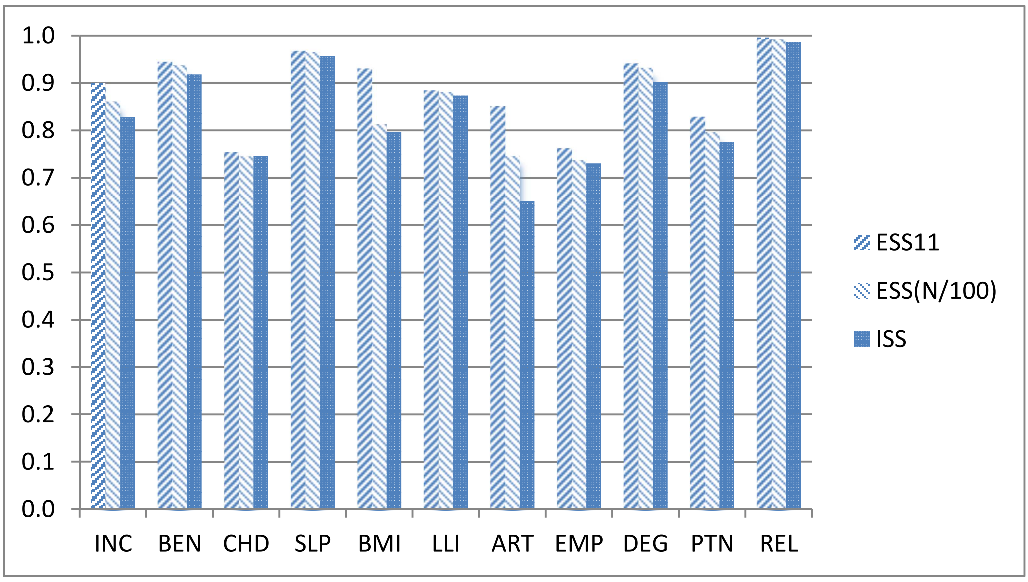
The other variable that stands out in Figure 1 is the only attitudinal variable in the study, the proportion of people agreeing with the statement that religion makes a big difference in life. This variable stands out because the precision gains from stratification are much more modest than for all other variables. Beliefs about the importance of religion are only very weakly associated with age.

Turning now to the final column of Table 2, it can be seen that the consecutive pairs method of variance estimation for ISS results in a modest over-estimation of standard errors, i.e. an under-estimation of the precision gain from stratification. The expected value of the estimated standard error is typically similar to, or just slightly smaller than, the true standard error with the ESS(N/100) design. This is of course the design that is assumed by expression (4) with $n_j = 2$, but the estimated standard errors differ from the true standard errors under this design due to the data having been generated by a different mechanism.

Table 2: Standard errors under four sample designs, and mean estimated standard errors for implicit stratified sampling

	Mean	s.e.				Est.(s.e.)
Continuous variables		SRS	ESS(11)	ESS(N/100)	ISS	ISS
Total monthly income	1479.0	49.82	47.28	46.22	45.33	46.24
Monthly benefit income	466.0	37.28	36.23	36.08	35.72	36.15
Number of children	1.600	0.0467	0.0406	0.0403	0.0403	0.0403
Hours of sleep	6.97	0.0587	0.0577	0.0576	0.0574	0.0576
Body mass index	26.06	17.03	16.42	15.35	15.19	15.28
Binary variables						
Limiting long-term illness (%)	34.93	1.489	1.400	1.397	1.392	1.396
Arthritis (%)	14.29	1.130	1.042	0.976	0.912	0.968
In paid employment (%)	52.29	1.560	1.362	1.339	1.333	1.339
Has degree (%)	21.37	1.281	1.242	1.236	1.216	1.226
Lives with spouse/partner (%)	61.51	1.520	1.384	1.356	1.338	1.344
Religion makes a great difference (%)	22.13	1.340	1.337	1.334	1.331	1.333

Figure 1: Design effects for three sample designs



5. DISCUSSION

The simulation study has shown, using real survey data, that ISS provides useful precision gains relative to ESS. This is true even when comparing to the most detailed form of ESS possible, namely that which involves creating strata such that just two selections are made from each stratum (i.e. the minimum number that permits variance estimation.) This result should lead researchers to question why, whenever useful auxiliary data are available for sample stratification, one would ever choose not to use implicit stratification, given that estimates will be less precise as a result. In practice, ESS typically involves a rather smaller number of strata, such that the average number of sample elements selected from each stratum is very considerably greater than two, perhaps more akin to the ESS11 design presented here. In this study, the ISS design produced substantially smaller standard errors than the ESS11 design, so there seems to be a strong case for ISS designs rather than ESS designs of this kind.

Furthermore, the approximation that must be used to estimate standard errors with ISS results in only a modest over-estimation. This would make statistical tests slightly conservative, which is probably more desirable than the false precision that would be provided by the opposite. In any case, the extent of the over-estimation (systematic error) is most likely small compared to the extent of sampling variance in the standard error estimate (random error).

The choice between ESS and ISS would therefore seem to come down to a choice between improved precision of the survey estimate or unbiased estimation of the precision of the survey estimate. To take the estimation of the proportion of people suffering from arthritis as a concrete example, would researchers prefer to have a standard error of 1.130 associated with their estimate (expected value) of 14.29 and to have an estimate of the standard error with an expected value of 1.130, or to have a standard error of 0.912 and an estimate of the standard error with an expected value of 0.968?

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